Markus Weimar
Ruhr-Universität Bochum, Germany

Explicit regularity estimates for solutions to quasi-linear PDEs

It is well-known that the rate of convergence of numerical schemes which aim to approximate solutions to operator equations is closely related to the maximal regularity of these solutions in certain scales of smoothness spaces of Sobolev and Besov type. For linear elliptic PDEs on Lipschitz domains, a lot of results in this direction already exist. In contrast, it seems that not too much is known for nonlinear problems.

In this talk, we are mainly concerned with the p-Laplace operator which has a similar model character for quasi-linear equations as the ordinary Laplacian for linear problems. It finds applications in models, e.g., for turbulent flows of a gas in porous media, radiation of heat, as well as in non-Newtonian fluid theory.
We discuss a couple of local regularity estimates for the gradient of the unknown solutions. These assertions are then used to derive global smoothness properties by means of wavelet-based proof techniques. Finally, the presented results imply that adaptive algorithms are able to outperform (at least asymptotically) their commonly used counterparts based on uniform refinement.

The material extends assertions which were obtained earlier in joint work with S. Dahlke, L. Diening, C. Hartmann, and B. Scharf (Nonlinear Anal., 130:298-329, 2016).