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High-dimensional approximation and sparse FFT

We consider the approximate reconstruction of a high-dimensional (e.g. $d = 10$) function from samples using trigonometric polynomials $p_I \colon \mathbb{T}^d \simeq [0,1)^d \to \mathbb{C}$,

$$p_I(\boldsymbol{x}) := \sum_{\boldsymbol{k}\in I} \hat{p}_{\boldsymbol{k}} \, \mathrm{e}^{2\pi\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}, \quad \hat{p}_{\boldsymbol{k}} \in \mathbb{C},$$

where $I \subset \mathbb{Z}^d$ is a suitable frequency index set, $|I| < \infty$. As spatial discretization, we use so-called reconstructing rank-1 lattices. Especially, we deal with the case where the frequency index set I is unknown. For this setting, we present a method which adaptively constructs the index set I of frequencies belonging to the approximately largest Fourier coefficients in a dimension incremental way. This method computes projected Fourier coefficients from samples along suitable rank-1 lattices $\Lambda(\boldsymbol{z}, M) := \{\frac{j}{M}\boldsymbol{z} \mod \boldsymbol{1} \colon j = 0, \ldots, M-1\} \subset \mathbb{T}^t$, $\boldsymbol{z} \in \mathbb{Z}^t$, $t \in \{1, \ldots, d\}$, and then determines the frequency locations. For the computation, only one-dimensional fast Fourier transforms (FFTs) and simple index transforms are used. We discuss an extension of this method which uses so-called multiple reconstructing rank-1 lattices such that the number of required samples and arithmetic operations is distinctly reduced. We demonstrate the high performance of the proposed methods in several numerical examples.

This is joint work with Lutz Kämmerer and Daniel Potts.