Felix Voigtlaender
TU Berlin, Germany

Structured Banach frame decompositions of decomposition spaces

We present a recent theory which implies existence of particularly nice **Banach frames** and **atomic decompositions** for **decomposition spaces**. Specifically, the resulting systems are *generalized shift-invariant systems* that are usually obtained from a *finite* set of *compactly supported* generators via certain translations, modulations and dilations.

The decomposition spaces $\mathcal{D}(\mathcal{Q}, L^p, \ell_w^q)$ mentioned above are defined similarly to Besov spaces, but the usual dyadic covering is replaced by an (almost) arbitrary **covering** $\mathcal{Q}$ of the frequency domain. In particular, the class of decomposition spaces includes homogeneous and inhomogeneous **Besov spaces** and **(α)-modulation spaces**.

In addition to the decomposition spaces $\mathcal{D}(\mathcal{Q}, L^p, \ell_w^q)$, the covering $\mathcal{Q}$ also determines a mapping $\psi \mapsto \Psi_\delta$ which associates a certain *generalized shift invariant system* $\Psi_\delta = \Psi_\delta(\psi)$ to a finite set ψ of **generators**. The parameter $\delta > 0$ determines the **sampling density** of this system. Specific examples include **Gabor systems** (if $\mathcal{Q}$ is the uniform covering) and **wavelet systems** (if $\mathcal{Q}$ is the dyadic covering).

The presented theory yields certain conditions concerning the generators ψ, such that if these conditions are satisfied, then the family $\Psi_\delta(\psi)$ *simultaneously* forms a Banach frame and an atomic decomposition for a whole *range* of decomposition spaces, for sufficiently fine sampling density δ. These conditions concerning the generators are quite technical in general, but they reduce to *readily verifiable and natural* conditions (smoothness, decay, vanishing moments) for most special cases.

Although the presented theory has many similarities to (generalized) **coorbit theory**, it is applicable in some relevant cases where coorbit theory is not known to apply. In particular, the theory can be used to establish equivalence of **analysis sparsity** and **synthesis sparsity** with respect to (sufficiently nice) cone-adapted **shearlet systems**, a novel result with relevant applications concerning the approximation of **cartoon-like functions**.