

Alexander Tyulenev

Steklov Mathematical Institute of Russian Academy of Sciences, Russia

Whitney-type Problem for Sobolev spaces on d -thick sets

Let $S \subset \mathbb{R}^n$ be a closed set such that for some $d \in [0, n]$ and $\varepsilon > 0$ the d -Hausdorff content $\mathcal{H}_\infty^d(S \cap B) \geq \varepsilon r^d$ for any ball B centers in S of volume $|B| \leq 1$. Let $p \in (\max\{1, n - d\}, \infty)$, $q \in (\max\{1, n - d\}, p)$, $l \in \mathbb{N}$ and $W_p^m(\mathbb{R}^n, \gamma)$ be the Sobolev space with Muckenhoupt weight $\gamma \in A_{\frac{p}{q}}(\mathbb{R}^n)$. We give an intrinsic characterization of the restrictions $\{D^\alpha F|_S : |\alpha| \leq l - 1\}$ to S of $(m - 1)$ -jets generated by functions $F \in W_p^m(\mathbb{R}^n, \gamma)$. In particular, for $p > n - 1$ we characterize the trace space of the classical Sobolev space $W_p^1(\mathbb{R}^n)$ to the closure $\bar{\Omega}$ of arbitrary path-connected set Ω . The results obtained in the present paper supplement the available results in which a similar problem was considered either in the case $p \in (n, \infty)$ without constraints on a set S or in the case $p \in (1, \infty)$ with much more stringent restrictions on a set S .