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Approximation Numbers of Embeddings of Sobolev Spaces of Dominating Mixed Smoothness – Preasymptotics and Asymptotics

We investigate the approximation of d-variate periodic functions in anisotropic Sobolev spaces of dominating mixed (fractional) smoothness $\vec{s}$ on the d-dimensional torus, where the approximation error is measured in the L_2-norm.
As it is well-known, in high dimensions functions from isotropic Sobolev spaces $H^s(\mathbb{T}^d)$ can not be approximated sufficiently fast (in the sense of approximation numbers of corresponding embeddings). One needs to switch to smaller spaces. Since the beginning of the sixties it is known that Sobolev spaces of dominating mixed smoothness $H^s_{\mathrm{mix}}(\mathbb{T}^d)$ may help. However, for very large dimensions even these classes are oversized. A way out is to sort the variables in dependence of there importance. This can be done in various ways. Here we make use of the following approach. We associate to each variable different smoothness assumptions. As smoother the function is with respect to the variable x_ℓ as weaker is the influence of this variable. This philosophy is reflected in the choice of the function space $H^{\vec{s}}_{\mathrm{mix}}(\mathbb{T}^d)$ characterized by the norm

$$\| f \,|H^{\vec{s}}_{\mathrm{mix}}(\mathbb{T}^d)\| := \Big[\sum_{k\in\mathbb{Z}} |c_k(f)|^2 \prod_{j=1}^{d} \big(1+|k_j|\big)^{2s_j} \Big]^{1/2} < \infty \,.$$

We assume $\vec{s} = (s_1, s_2, \ldots , s_d)$ and

$$s_1 = s_2 = \ldots = s_\nu < s_{\nu+1} \le s_{\nu+2} \ldots \le s_d$$

for some number $1 \le \nu < d$. It will be the main aim of my talk to describe the behaviour of the approximation numbers

$$a_n(I_d : \; H^{\vec{s}}_{\mathrm{mix}}(\mathbb{T}^d) \to L_2(\mathbb{T}^d))$$

in dependence of $n, \vec{s}, \nu$ and d.
This is joined work with Thomas Kühn (Leipzig) and Tino Ullrich (Bonn).