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Embeddings and characterizations of function spaces with logarithmic smoothness

Function spaces of logarithmic smoothness are useful to get the complete solution of some natural questions such as limiting embedding theorems, fractal analysis and a related spectral theory, probability theory and the theory of stochastic processes. The aim of this talk is to go deeply into the relationships between function spaces involving logarithmic smoothness. Namely, we study embeddings between

- Besov spaces given by differences $\mathbb{B}^{s,b}_{p,q}$.
- Fourier-analytically defined Besov spaces $B^{s,b}_{p,q}$.
- Triebel-Lizorkin spaces $F^{s,b}_{p,q}$.
- Bessel potential spaces $W^{s,b}_{p}$.

Here, b is the exponent of an additional logarithmic smoothness. Special emphasis is given to the delicate case $s = 0$, where it is known that equivalent approaches to introduce smoothness spaces with $s > 0$ necessarily differ in this limiting situation. To overcome this obstruction we apply a shift in the logarithmic smoothness. In particular, it turns out that $\mathbb{B}^{0,b}_{2,2} = W^{0,b+1/2}_{2}$. We also derive characterizations of the above mentioned function spaces in terms of the Fourier transform/Fourier coefficients for general monotone functions and lacunary Fourier series. These descriptions allow us to establish the sharpness of the embedding theorems. Our approach relies on limiting interpolation, extension theorems and duality arguments.
This is joint work with S. Tikhonov (Barcelona).