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**Inequalities in approximation theory involving derivatives
of functions in L_p , $0 < p < 1$**

Several new inequalities for moduli of smoothness and errors of the best approximation of a function and its derivatives in the spaces L_p , $0 < p < 1$, are obtained. For example, it is shown that for any $0 < p < 1$ and $k, r \in \mathbb{N}$

$$\omega_{r+k}(f, \delta)_p \leq C(p, k, r) \delta^{r+\frac{1}{p}-1} \left(\int_0^\delta \frac{\omega_k(f^{(r)}, t)_p^p}{t^{2-p}} dt \right)^{\frac{1}{p}},$$

where the function f is such that $f^{(r-1)}$ is absolutely continuous. Similar inequalities are obtained for the Ditzian-Totik moduli of smoothness and the error of the best approximation of functions by trigonometric and algebraic polynomials and splines. As an application, positive results about simultaneous approximation of a function and its derivatives by the mentioned approximation methods in the spaces L_p , $0 < p < 1$, are derived.