Anna Kamont
Institute of Mathematics, Polish Academy of Sciences, Gdańsk, Poland

Directional wavelet projections and interpolatory estimates for Riesz transform revisited

The problem presented in this talk has its origin in papers [4], [2], [3]. In these papers, the authors obtained the following interpolation inequality: let $\Phi \in L^2(R^d)$ be a fixed element of the Haar system on R^d, or a fixed element of an orthonormal wavelet system on R^d, satisfying some Hölder condition, and let $P_\Phi u = \sum_{j\in Z, k\in Z^d}(u, \Phi_{j,k})\Phi_{j,k}$ be the orthogonal projection onto the space spanned by $\{\Phi_{j,k}(\cdot) = 2^{dj/2}\Phi(2^j \cdot -k), j \in Z, k \in Z^d\}$. Then the following inequality holds:

$$(*) \qquad \|P_\Phi u\|_p \le C\|u\|_p^{1-\theta}\|R_i u\|_p^{\theta}, \quad 1 < p < \infty,$$

where R_i is Riesz transform on R^d in i-th direction, and $0 < \theta < 1$ is an exponent, depending only on the wavelet under consideration and p. In case of the Haar wavelet, there is $\theta = 1/2$ for $2 \le p < \infty$ and $\theta = 1 - 1/p$ for $1 < p \le 2$, and these exponents are best possible. In case of a wavelet satisfying the Hölder condition with an exponent $0 < \alpha < 1$, there is $\theta = \alpha$ for all p; there is also a version of $(*)$ for $\alpha = 1$.
The aim of this talk is to explain the nature of the exponent θ appearing in $(*)$ in case $p = 2$ for more general functions Φ. For $\Phi \in L^2(R^d)$ such that $\{\Phi_{j,k}(\cdot) = 2^{dj/2}\Phi(2^j \cdot -k), j \in Z, k \in Z^d\}$ is an orthonormal system in $L^2(R^d)$, we describe exponents θ in $(*)$ in terms of coefficients of an expansion of Φ with respect to a suitable wavelet basis. This in turn allows us to formulate some necessary condition and some sufficient condition for $(*)$ in terms of regularity of Φ.
The talk is based on paper [1], joint with Paul F.X. Müller (J.Kepler University, Linz, Austria).

References.

[1] A. Kamont, P.F.X. Müller, Directional wavelet projections and interpolatory estimates for Riesz transform revisited. In preparation

[2] J. Lee, P.F.X.Müller, S.Müller, Compensated compactness, separately convex functions and interpolatory estimates between Riesz transforms and Haar projections. Comm. Partial Differential Equations, 36 (4), 547 – 601, 2011.

[3] P.F.X. Müller, S. Müller, Interpolatory estimates, Riesz Transforms and Wavelet Projections. Revista Math. Iberoam. 32 (4), 1137 – 1162, 2016.

[4] S. Müller, Rank-one convexity implies quasiconvexity on diagonal matrices. Internat. Math. Res. Notices 20, 1087 – 1095, 1999.