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## Poincaré inequalities and compact embeddings from Newtonian Sobolev spaces into weighted $L^q$-spaces

Assume that $X = (X, d)$ is a complete metric space equipped with a metric $d$ and two positive locally finite Borel measures $\mu$ and $\nu$. We discuss compactness and continuity of embeddings from Newtonian - Sobolev spaces defined on metric spaces into $L^q$-spaces with respect to possibly another measure. We do not assume that the involved measures are doubling and so we cannot exploit typical scalling properties. Instead, our results are formulated in a possibly general form, using covering families and local Poincaré type inequalities. For example, our main idea for compactness is to construct countably many two-weighted local Poincaré inequalities involving measures $\mu$ and $\nu$, which are satisfied on certain covering sets $E_i$ and $E'_i$:

$$\left(\int_{E_i} |u - a_{E_i}(u)|^q \, d\nu\right)^{1/q} \leq C(r)\left(\int_{E'_i} g^p \, d\mu\right)^{1/p},$$

whenever $u$ belongs to the apropriate Newtonian–Sobolev space and $g$ is a $p$-weak upper gradient of $u$.
We show how to construct such suitable coverings and illustrate our tools on concrete examples. Among other results, we recover several classical embedding theorems on domains and fractal sets in $\mathbf{R}^n$. Results are obtained together with Jana Björn.