Mikhail L. Gol'dman
Peoples Friendship University Russia, Moscow, Russia

Estimates for monotone operators on the cone of decreasing functions in weighted Oricz spaces

Let $R_+ = (0,\infty)$, $M = M(R_+)$ be the set of Lebesgue measurable functions, and $M_+ = \{f \in M(R_+) : f \geq 0\}$, $\dot{M}_+ = \{v \in M(R_+) : 0 < v < \infty\}$. Let $\Phi : [0,\infty) \to [0,\infty)$ be an N-function, see [1]. For $f \in M, v \in \dot{M}_+$ denote

$$\|f\|_{\Phi,v} := \inf\left\{\lambda > 0 : \int_0^\infty \Phi\left(\lambda^{-1}|f(x)|\right) v(x)dx \leq 1\right\}.$$

The Orlicz space $L_{\Phi,v}$ is determined as the set of functions $f \in M$ with $\|f\|_{\Phi,v} < \infty$. Assume that $0 < V(t) := \int_0^t v d\tau < \infty, \quad \forall t \in R_+, \quad V(+\infty) = \infty$. Let $u, v, w \in \dot{M}_+, \Phi_1, \Phi_2$ be N-functions. We study the modular inequality:

$$\exists c_1 \in R_+ : \Phi_2^{-1}\left\{\int_{R_+} \Phi_2\left(wTf\right) udt\right\} \leq \Phi_1^{-1}\left\{\int_{R_+} \Phi_1(c_1 f) vdt\right\}, \quad f \in \Omega, \tag{1}$$

for the operator $T(f;t) = \int_0^t f(\tau)d\tau,\ t \in R_+$ on the cone $\Omega = \{f \in L_{\Phi,v} : 0 \leq f \downarrow\}$. We assume that $\Phi_1 \prec \Phi_2$, i.e. $\exists\, C_0 \in R_+ :$ for every nonnegative sequence $\{a_i\}$

$$\sum_i \Phi_2 \circ \Phi_1^{-1}(a_i) \leq C_0\ \Phi_2 \circ \Phi_1^{-1}(\sum_i a_i).$$

Note that if $\Phi_2 \circ \Phi_1^{-1}$ is convex, then $\Phi_1 \prec \Phi_2$ with $C_0 = 1$.
Under above assumptions the necessary and sufficient conditions are found on modular functions Φ_1, Φ_2 and weights u, v, w for the validity of inequality (1); see [2].

This work is supported by the Russian Scientific Foundation (projects no. 14-11-00443).

References.

[1] C. Bennett, R. Sharpley, *Interpolation of operators,* Pure Appl. Math., **129**, Acad. Press, Boston, 1998.

[2] E. G. Bakhtigareeva, M. L. Goldman, Mathematical Notes, 2017, to appear.