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Hardy spaces associated with semi-groups of linear operators

Let $(X, d(x, y))$ be a metric space equipped with a nonnegative measure μ and let $\{T_t\}_{t>0}$ be a semigroup of linear operators generated by $-L$ acting on $L^p(X)$. We say that f belongs to the Hardy space H^1_L if the maximal function

$$\mathcal{M}_L f(x) = \sup_{t>0} |T_t f(x)|$$

belongs to $L^1(X, \mu)$. Then we set $\|f\|_{H^1_L} = \|\mathcal{M}_L f\|_{L^1(X,\mu)}$. These spaces are extensions of the classical Hardy spaces which can be thought as those associated with the classical heat semigroup $T_t = e^{t\Delta}$. Examples of such semigroups which are subject of our interest are those generated by Schrödinger operators, Dunkl operators, Grushin operators. During the talk we shall discuss properties of the Hardy spaces, including their atomic decompositions and Riesz transform characterizations. If time permits we shall also consider boundedness of spectral multiplier operators on the Hardy spaces.
The results are joint works with J-Ph. Anker, N. Ben Salem, N. Hamda, M. Preisner, B. Wróbel, and J. Zienkiewicz.