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Entropy and approximation numbers of embeddings of spaces of block-radial functions

Sobolev embeddings

$$B^{s_1}_{p_1,q_1}(\mathbb{R}^d) \hookrightarrow B^{s_2}_{p_2,q_2}(\mathbb{R}^d),$$

where $B^s_{p,q}(\mathbb{R}^d)$ are Besov spaces, are never compact. Therefore if we want to investigate asymptotic behaviour of entropy and approximation numbers of this embeddings than we need to modify the Besov spaces. Today we have good knowledge about embeddings of radial Besov spaces. However we can extend the study on more general spaces, so-called block-radial spaces.
Let $d = \gamma_1 + \ldots + \gamma_m$ and $SO(\gamma) = SO(\gamma_1) \times \ldots \times SO(\gamma_m)$ be a group of isometries on decomposition of $\mathbb{R}^d = \mathbb{R}^{\gamma_1} \times \cdots \times \mathbb{R}^{\gamma_m}$ with the action of $g = (g_1, \ldots, g_m) \in SO(\gamma)$ defined by

$$g(\tilde{x}_1, \ldots, \tilde{x}_m) = (g_1(\tilde{x}_1), \ldots, g_m(\tilde{x}_m)), \qquad \tilde{x}_i \in \mathbb{R}^{\gamma_i}.$$

By $R_\gamma B^s_{p,q}(\mathbb{R}^d)$ we mean the subset of $SO(\gamma)$-invariant distributions (functions) in $B^s_{p,q}(\mathbb{R}^d)$ and we endow this subset with the same norm as the original space.
The embedding

$$R_\gamma B^{s_1}_{p_1,q_1}(\mathbb{R}^d) \hookrightarrow R_\gamma B^{s_2}_{p_2,q_2}(\mathbb{R}^d) \tag{1}$$

is compact if and only if $p_1 < p_2, s_1 - \frac{d}{p_1} - s_2 + \frac{d}{p_2} > 0$ and $\min_i \gamma_i \geq 2$.
Furthermore we can reduce investigation of the asymptotic behaviour of the entropy and the approximation numbers of the embeddings (1) to the estimation for embeddings of corresponding weighted spaces with the Muckenhoupt weight. Moreover using the wavelet characterization of Besov spaces with $\mathcal{A}_\infty$ weights we can use the technique of discretization i.e., we can reduce the problem to the corresponding problem for suitable sequence spaces.