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Sparse approximation with respect to the Faber-Schauder system

We consider approximations of multivariate functions using m terms from its tensorized Faber-Schauder expansion. The univariate Faber-Schauder system on $[0,1]$ is given by dyadic dilates and translates (in the wavelet sense) of the L_∞ normalized simple hat function with support in $[0,1]$. We obtain a hierarchical basis which will be tensorized over all levels (hyperbolic) to get the dictionary $\mathcal{F}$. The worst-case error with respect to a class of functions $\mathbf{F} \hookrightarrow X$ is measured by the usual best m-term widths denoted by $\sigma_m(\mathbf{F},\mathcal{F})_X$, where the error is measured in X. We prove upper founds for $\sigma_m(\mathbf{F},\mathcal{F})_{L_p([0,1]^d)}$ in case of Sobolev and Besov spaces. Especially we constructively prove the following sharp asymptotic bound for the class of Besov spaces with small mixed smoothness (i.e. $1/p < r < \min\{1/\theta - 1, 2\}$) in L_∞

$$\sigma_m(S^r_{p,\theta}B,\mathcal{F})_\infty \asymp m^{-r}\,.$$

Note, that this asymptotic rate of convergence does not depend on the dimension d (only the constants behind). In addition, the error is measured in L_∞ and to our best knowledge this is the first sharp result involving L_∞ as a target space. We emphasize two more things. First, the selection procedure for the coefficients is a level-wise constructive greedy strategy which only touches a finite prescribed number of coefficients. And second, due to the use of the Faber-Schauder system, the coefficients are finite linear combinations of discrete Function values. Hence, this method can be considered as a nonlinear adaptive sampling algorithm leading to a pure polynomial rate of convergence for any d.

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